Comparison of Statistical Models for the Lumen Lifetime Distribution of High Power White LEDs

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Abstract— Compared to conventional light sources, high power white LED (HPWLED) possesses superior benefits in terms of efficiency, power consumption, environmental friendliness, and lifetime. Therefore, the market of HPWLED is growing rapidly in the application of general lighting, LCD-TVs backlighting, motor vehicle lighting. However, traditional reliability assessment techniques have several limitations on this highly reliable electronic device with little failure during life test. This paper uses the general degradation path model to analyze the lumen maintenance data of HPWLED with two approaches (Approximation approach and Analytical approach). And three statistical models (Weibull, Lognormal, and Normal) were utilized to predict the lumen lifetime of HPWLED and finally the prediction results were verified by the Akaike Information Criterion (AIC). Results show that Weibull model is the best-fitting one to the “pseudo failure time” data in the approximate approach, however, Lognormal is the most suitable fitting model for the random effect parameter, $\beta$, in analytical approach.

Keywords— High Power White LEDs, Lumen Lifetime Distribution, Statistical Models, Weibull Distribution, Lognormal Distribution, Normal Distribution, Akaike Information Criterion (AIC)

I. INTRODUCTION

Since J. Holonyak and S. Bevacqua [1] invented the first light-emitting diodes (LEDs) in 1962, LEDs have made remarkable development over the past half century in the application of street lighting, display backlighting, signage and general luminaries [2]. As one of the most potential alternates of traditional lighting sources (such as Incandescent Lamp, and Cold Cathode Fluorescent Lamp (CCFL)), High Power White Light-emitting diode (HPWLED) has attracted increasing interest in the field of lighting systems owing to their high efficiency, environmental benefits and long lifetime in applications [2]. However, also due to its longer lifetime, higher reliability, and its different failure mechanisms compared to traditional light sources, there has been no standard method to evaluate and predict the reliability of HPWLED until now. Therefore, how to predict the lifetime accurately for such highly reliable electronic product is becoming a key issue in popularizing this novel device in the LED lighting market.

From the previous failure mode and failure mechanism analysis results [3-5], lumen degradation is one of the most critical failures in the HPWLED module and sometimes lumen lifetime can be characterized as LEDs life in some application areas. Thus, predicting the remaining useful lumen life is the important procedure to assess the reliability of HPWLED for LEDs manufactures.

Previously, Lu and Meeker [6] proposed a method for modeling the degradation data, called a ‘general degradation path model’ which modeled the degradation as a function of time and multidimensional random variables. After that, M.A. Freitas et al [7] applied this model to the train wheel data and assessed its reliability. Different from wheel linear degradation path, HPWLED’s lumen degradation always follows nonlinear path; and little research has been applied this general degradation path model to assess the reliability of HPWLED.

This paper uses the general degradation path model to analyze the lumen maintenance data of HPWLED with two approaches: a) approximation approach, and b) analytical approach. And three statistical models: a) Weibull, b) Lognormal, and c) Normal were utilized to predict the lumen lifetime of HPWLED. Finally, the prediction results were verified by the Akaike Information Criterion (AIC).

II. TEST DEVICE AND TEST CONDITION DESCRIPTION

In this paper, White LUXEON Rebel, one of highly reliable HPWLEDs with high luminous flux (>100 lumens in cool white at 350mA) from PHILIPS, was chosen as the research object [8].

![Figure 1 White LUXEON Rebel (source from LUMILEDs, PHILIPS [8])](image)

<table>
<thead>
<tr>
<th>TABLE I EES LM-80-08 TEST CONDITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM-80-08 Test Temperature</td>
</tr>
<tr>
<td>55 °C</td>
</tr>
</tbody>
</table>

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And the lumen maintenance data under one test condition (Table I) were selected from DR03: LM-80 Test Report [9], which was collected according to the standard test procedure proposed by Illumination Engineering Society (IES LM-80-08, Approved Method for Lumen Maintenance Testing of LED Light Source [10]). The data set is shown in Table II.

### III. THEORY AND METHODOLOGY

#### A. General degradation path model

The general degradation path model was first presented by Lu and Meeker [6]. It supposed that, a random sample size is supposed as n, and the measurement times are \( t_1, t_2, t_3, \ldots, t_n \). The performance measurement for the \( i^{th} \) unit at the \( i^{th} \) test time is referred to as \( y_{ij} \). So the degradation path can be registered as the time-performance measurement pairs \((t_i, y_{ij}), (t_{i2}, y_{ij2}), \ldots, (t_{imin}, y_{ijmin})\), for \( i = 1, 2, \ldots, n \), and \( m_i \) represents the test time points for each unit

\[
y_{ij} = D(t_i; \alpha, \beta_i) + \epsilon_{ij}
\]

where \(D(t_i; \alpha, \beta_i)\) is the actual degradation path of unit \( i \) at the measurement time \( t_i \), \( \alpha \) is the vector of fixed effects which remain constant for each unit, \( \beta_i \) is a vector of random effects, which vary according to the diverse material properties of the different units and their production processes or handing conditions. \( \epsilon_{ij} \) represents the measurement errors for the unit \( i \) at the time \( t_i \), which is supposed to be a normal distribution with zero mean and constant variance, \( \epsilon_{ij} \sim \text{Normal}(0, \sigma^2) \).

![Figure 2 General degradation path model (a) increasing type; (b) decreasing type](image)

Failure definition for the general degradation path models is that the performance measurement \( y_{ij} \) exceeds (or is lower than) the critical threshold \( D_j \) at time \( t \). And pdf is the probability density failure distribution of sample. The cumulative probability of failure function \( F(t) \) is given as follows (Figure 2):

The increasing type of performance measurement

\[
F(t) = P(t \leq T) = P[D(t_i, \alpha, \beta_i) \leq D_j]
\]

Time to Failure \( T = \inf(t \geq 0; D(t_i, \alpha, \beta_i) \geq D_j) \)

The decreasing type of performance measurement

\[
F(t) = P(t \leq T) = P[D(t_i, \alpha, \beta_i) \leq D_j]
\]

Time to Failure \( T = \inf(t \geq 0; D(t_i, \alpha, \beta_i) \leq D_j) \)

To estimate the time to failure distribution, \( F(t) \), based on the degradation data, two statistical approaches (the approximation approach and the analytical approach) were used in this paper. And it can be summarized into two basic steps: (1) estimating the parameters for degradation path model (2) evaluating the time to failure distributions, \( F(t) \).

In details, The LEDs empirical lumen degradation path model can be expressed as follows:

\[
y_{ij} = D(t_i; \alpha; \beta) = \alpha \cdot \exp(-\beta \cdot t_i)
\]

where \( y_{ij} \) represents the lumen maintenance data at each test time \( t_i \) (Table II), \( \alpha \) is initial constant, and \( \beta \) is the degradation rate. According to the IES LM-80-08 standard [10], the critical failure threshold \( D_j \) is defined as lumen lifetime \( L_{20} \), which means that the lumen output decreases to the 70 percents of the initial one over a certain length of operation time.

1) Approximation approach [7]

The approximation method predicts each unit’s time to failure based on the general degradation model and projects to the “pseudo” failure time when the degradation path reaches the critical failure threshold, \( D_j \). Normally, the steps of the analysis are as follows:

(a) Use the Nonlinear Least Squares (NLS) method to estimate the parameters (fixed effect parameter \( \alpha \) and random effect parameter \( \beta_i \)) of degradation path model, based on the measured data \((t_{ij}, y_{ij}), (t_{i2}, y_{ij2}), \ldots, (t_{imin}, y_{ijmin})\) for each unit, and the estimated results are \( \alpha \) and \( \beta_i \) respectively.

(b) Extrapolate the degradation path model of each unit to critical failure threshold, \( D_j \). When \( D(t_{ij}; \alpha; \beta_i) = D_j \), the “pseudo failure (not real failures) time” for each unit \((t_{ij}, t_{i2}, \ldots, t_{iL})\) can be predicted.

(c) Fit the probability distribution for these “pseudo life time” data and estimate the associated parameters for each distribution with maximum likelihood estimation (MLE) method.

(d) Get the reliability function, \( R(t) \).

2) Analytical approach

Regarding simple degradation path models, researchers found that there were some relationships between the random parameters of degradation path models and cumulative probability of failure distribution \( F(t) \) [11]. Therefore, the reliability information of the sample could be obtained through analyzing the statistical properties of random effects parameters \( \beta_i \).

(a) The first step of the analytical method is also to estimate the parameters (fixed effect parameter, \( \alpha \), and random effect parameter, \( \beta_i \)) using the NLS method for each unit, like the first step of the approximate method.

(b) Infer the cumulative probability of failure distribution, \( F(t) \), based on the statistical properties of random effect parameters \( \beta_i \). For the LED lumen degradation, there is only one random parameter, \( \beta \), as shown in the equation (4). In this paper, we study the cases that \( \beta \) follows the three types of statistical models (Weibull, Lognormal, Normal) respectively.
Case 1 The random effect parameter $\beta$ follows the two-parameter Weibull distribution with shape parameter $\delta_\beta$ and scale parameter $\lambda_\beta$. The probability density function of $\beta$ is given by

$$f_\beta(x) = \delta_\beta \left( \frac{x}{\lambda_\beta} \right)^{\delta_\beta - 1} \exp \left[- \left( \frac{x}{\lambda_\beta} \right)^{\delta_\beta} \right]$$

(5)

And the cumulative distribution function (CDF) of $\beta$ is:

$$F_\beta(x) = 1 - \exp \left[- \left( \frac{x}{\lambda_\beta} \right)^{\delta_\beta} \right]$$

(6)

When the lumen performance researches critical failure threshold, $D_t$ is equal to $\alpha \exp(-\beta \cdot T)$, the time to failure, $T$, can be expressed as:

$$T = \frac{\ln(D_t / \alpha)}{-\beta}$$

(7)

The CDF of time to failure, $F(t)$ is:

$$F(t) = P(t \leq T) = P \left[ t \leq \frac{\ln(D_t / \alpha)}{-\beta} \right] = P \left[ \beta \leq \frac{\ln(D_t / \alpha)}{-t} \right]$$

$$= 1 - \exp \left[- \left( \frac{\ln(D_t / \alpha)}{-t} \right)^{\delta_\beta} \right]$$

(8)

In this situation, the reciprocal of time to failure, $1/T$, is also Weibull distribution with shape parameter $\delta_{1/T} = \delta_\beta$, scale parameter $\lambda_{1/T} = \lambda_\beta / \ln(\alpha / D_t)$, $1/T$-Weibull ($\delta_{1/T}, \lambda_{1/T}$)-Weibull ($\delta_\beta, \lambda_\beta / \ln(\alpha / D_t)$). And the $p$-quantiles are given by:

$$t_p = \lambda_\beta^{-1} \ln(\alpha / D_t) \left[ \ln(1 - p) \right]^{-\delta_\beta}$$

(9)

Case 2 The random effect parameter $\beta$ follows the two-parameter Lognormal distribution with location parameter $\mu_\beta$ and scale parameter $\eta_\beta$. The probability density function of $\beta$ is given by

$$f_\beta(x) = \frac{1}{\eta_\beta x \sqrt{2\pi}} \exp \left[- \frac{(\ln x - \mu_\beta)^2}{2\eta_\beta^2} \right]$$

(10)

And the CDF of $\beta$ is:

$$F_\beta(x) = \Phi \left( \frac{\ln x - \mu_\beta}{\eta_\beta} \right)$$

(11)

Where $\Phi$ is the standard normal CDF.

Like the equation (8), $F(t)$ can be inferred by inserting the equation (7) - (10):

$$F(t) = P(t \leq T) = P \left[ t \leq \frac{\ln(D_t / \alpha)}{-\beta} \right] = P \left[ \beta \leq \frac{\ln(D_t / \alpha)}{-t} \right]$$

$$= \Phi \left( \frac{\ln t - \left[ \ln(\alpha / D_t) - \mu_\beta \right]}{\eta_\beta} \right)$$

(12)

This means that the time to failure, $T$, is also Lognormal distribution with location parameter $\mu_T = \ln(\ln(\alpha / D_t)) - \mu_\beta$ and scale parameter $\eta_T = \eta_\beta$. $T$-Lognormal ($\mu_T, \eta_T$) = Lognormal ($\ln(\ln(\alpha / D_t)) - \mu_\beta, \eta_\beta$). And the $p$-quantiles are given by

$$t_p = \exp \left[ z_p \eta_\beta + \ln[\ln(\alpha / D_t)] - \mu_\beta \right]$$

(13)

Where $z_p$ is the $p^{th}$ quantile of the standard normal distribution.

Case 3 The random effect parameter $\beta$ follows the Normal distribution with mean $\nu_\beta$ and standard deviation $\phi_\beta$, $\beta$-Normal ($\nu_\beta, \phi_\beta$). The probability density function of $\beta$ is given by

$$f_\beta(x) = \frac{1}{\phi_\beta \sqrt{2\pi}} \exp \left[- \frac{(x - \nu_\beta)^2}{2\phi_\beta^2} \right]$$

(14)

And the CDF of $\beta$ is:

$$F_\beta(x) = \Phi \left( \frac{x - \nu_\beta}{\phi_\beta} \right)$$

(15)

The $F(t)$ also can be given by

$$F(t) = P(t \leq T) = P \left[ t \leq \frac{\ln(D_t / \alpha)}{-\beta} \right] = P \left[ \beta \leq \frac{\ln(D_t / \alpha)}{-t} \right]$$

$$= \Phi \left( \frac{\ln(\alpha / D_t) - t \nu_\beta}{\phi_\beta} \right)$$

(16)

In this case, the reciprocal of time to failure, $1/T$, follows the Normal distribution with mean $\nu_{1/T} = \nu_\beta / \ln(\alpha/D_t)$ and standard deviation $\phi_{1/T} = \phi_\beta / \ln(\alpha/D_t)$. $1/T$-Normal ($\nu_{1/T}, \phi_{1/T}$) = Normal ($\nu_\beta / \ln(\alpha/D_t)$). And the $p$-quantiles are given by

$$t_p = \left( z_p \phi_\beta + \nu_\beta \right)^{-1} \ln(\alpha / D_t)$$

(17)

B. Akaike information criterion (AIC)

Akaike information criterion (AIC) is one method proposed by H. Akaike [12] to verify the goodness of fit of a proposed statistical model. The AIC is quantitatively defined as follows

$$AIC = 2\log(L) + 2 \cdot k$$

(18)

where $L$ is the maximum likelihood estimation (MLE) of the fitting model and $k$ is the number of independently adjusted parameters within the model. The judgment standard of this theory is to compare the AIC value of proposed fitting models and the lowest AIC value means the best model-fitting.

IV. RESULTS AND DISCUSSION

A. Lumen lifetime estimation with the approximation approach

Following the approximation method procedure shown in the above section, firstly, with the nonlinear least squares estimator, parameters of general degradation path model ($\alpha_i, \beta_i$) were estimated for each unit (Figure 3). And next, by extrapolating the model of each unit to the critical failure threshold (30% light decrease), “pseudo failure times” can be predicted (Table III). Thirdly, fitting the three types of statistical models (Weibull, Lognormal, Normal) for the “pseudo failure times” individually and estimating the associate parameters of each distributions with MLE method (list in Table IV).
Table III List of Estimated Parameters for Degradation Path Model and Pseudo Failure Time

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Pseudo failure time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.99858</td>
<td>4.57E-06</td>
<td>77816.5</td>
</tr>
<tr>
<td>2</td>
<td>1.01269</td>
<td>5.06E-06</td>
<td>72988.9</td>
</tr>
<tr>
<td>3</td>
<td>0.99518</td>
<td>3.66E-06</td>
<td>96138.6</td>
</tr>
<tr>
<td>4</td>
<td>1.00629</td>
<td>3.88E-06</td>
<td>93511.0</td>
</tr>
<tr>
<td>5</td>
<td>1.00624</td>
<td>4.28E-06</td>
<td>84788.7</td>
</tr>
<tr>
<td>6</td>
<td>0.99682</td>
<td>2.88E-06</td>
<td>122853.4</td>
</tr>
<tr>
<td>7</td>
<td>1.00572</td>
<td>3.22E-06</td>
<td>112472.9</td>
</tr>
<tr>
<td>8</td>
<td>1.01317</td>
<td>3.67E-06</td>
<td>100867.2</td>
</tr>
<tr>
<td>9</td>
<td>1.01559</td>
<td>4.42E-06</td>
<td>84287.7</td>
</tr>
<tr>
<td>10</td>
<td>1.0167</td>
<td>4.17E-06</td>
<td>89440.3</td>
</tr>
<tr>
<td>11</td>
<td>1.00823</td>
<td>3.34E-06</td>
<td>109122.0</td>
</tr>
<tr>
<td>12</td>
<td>1.00993</td>
<td>3.30E-06</td>
<td>111008.9</td>
</tr>
<tr>
<td>13</td>
<td>1.00618</td>
<td>2.90E-06</td>
<td>125152.1</td>
</tr>
<tr>
<td>14</td>
<td>1.00721</td>
<td>3.77E-06</td>
<td>96535.9</td>
</tr>
<tr>
<td>15</td>
<td>1.00053</td>
<td>3.52E-06</td>
<td>101485.3</td>
</tr>
<tr>
<td>16</td>
<td>1.00859</td>
<td>3.54E-06</td>
<td>103196.0</td>
</tr>
<tr>
<td>17</td>
<td>1.00537</td>
<td>3.29E-06</td>
<td>110333.4</td>
</tr>
<tr>
<td>18</td>
<td>1.0056</td>
<td>3.01E-06</td>
<td>120175.9</td>
</tr>
<tr>
<td>19</td>
<td>1.01107</td>
<td>3.23E-06</td>
<td>113760.5</td>
</tr>
<tr>
<td>20</td>
<td>1.00772</td>
<td>3.62E-06</td>
<td>100602.0</td>
</tr>
</tbody>
</table>

Table IV Estimated Parameters of Each Statistical Models by Approximate Approach

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Weibull</th>
<th>Lognormal</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta$</td>
<td>8.23059</td>
<td>(1.44514)</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>107.462</td>
<td>(3081.52)</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>11.5157</td>
<td>(0.0334102)</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.149415</td>
<td>(0.0245637)</td>
<td></td>
</tr>
<tr>
<td>$\nu$</td>
<td>101,317</td>
<td>(3274.03)</td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>14614.9</td>
<td>(2407.12)</td>
<td></td>
</tr>
<tr>
<td>Log $(L)$</td>
<td>-219.554</td>
<td>-220.171</td>
<td>-219.712</td>
</tr>
<tr>
<td>$k^*$</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>AIC</td>
<td>443.108</td>
<td>444.342</td>
<td>443.424</td>
</tr>
</tbody>
</table>

(·) is standard error of estimated parameters

$k^*$ is the independently adjusted parameters (2 for all three statistical models)
B. Lumen lifetime estimation using the analytical approach

As discussed in the section III, the key step of the analytical method is the probability analysis for random effect parameter, \( \beta \), by replacing the extrapolating step in the approximate. And the parameter, \( \alpha_i \), was supposed as fixed effect and equals to 1 as all lumen degradation data of each unit were normalized to 1 at the initial time.

![Figure 6 Reliability function prediction with Lognormal model-fitting](image)

![Figure 7 Reliability function prediction with Normal model-fitting](image)

Weibull reliability function:

\[
R(t) = \exp \left[ -\left( t / \lambda \right)^\delta \right] = \exp \left[ -\left( \frac{t}{107.462} \right)^{2.3059} \right] \tag{19}
\]

Lognormal reliability function:

\[
R(t) = 1 - \Phi \left( \frac{\ln t - \mu}{\eta} \right) = 1 - \Phi \left( \frac{\ln t - 11.5157}{0.149415} \right) \tag{20}
\]

Normal reliability function:

\[
R(t) = 1 - \Phi \left( \frac{t - \nu}{\varphi} \right) = 1 - \Phi \left( \frac{t - 101.317}{14614.9} \right) \tag{21}
\]

Based on the estimated parameters, three types of reliability functions are list from equation (19)-(21). By comparing of prediction results, we can conclude that: (1) all of three types of statistical models are fitted the pseudo failure time well; (2) according to the AIC value, Weibull model with lowest AIC value presents the best fitting performance among them.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Weibull</th>
<th>Lognormal</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>6.43168 (1.03806)</td>
<td>3.91805e-6 (1.44801e-7)</td>
<td>12.5277 (0.0343671)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>3.91805e-6 (1.44801e-7)</td>
<td></td>
<td>0.153694 (0.0252672)</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-12.5277</td>
<td>3.6665e-6 (1.30399e-6)</td>
<td>5.83616e-5 (9.58711e-5)</td>
</tr>
<tr>
<td>( \eta )</td>
<td>0.153694</td>
<td>0.153694</td>
<td></td>
</tr>
<tr>
<td>( \nu )</td>
<td>11.4968</td>
<td></td>
<td>1.0280e-5</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>1.0280e-5</td>
<td></td>
<td>1.6350e-6</td>
</tr>
<tr>
<td>Log (L)</td>
<td>257.77</td>
<td>260.132</td>
<td>259.217</td>
</tr>
<tr>
<td>AIC</td>
<td>-511.54</td>
<td>-516.264</td>
<td>-514.434</td>
</tr>
</tbody>
</table>

\( k \) is standard error of estimated parameters
\( k^* \) is the independently adjusted parameters (2 for all three statistical models)
The random effect parameters, $\beta$ of each test units were estimated and listed in Table III. And then like the approximate approach, three statistical models (Weibull, Lognormal, Normal) were also used to fit the $\beta$. The associate model parameters were estimated by the method of MLE and are shown in the Table V. According to the transformation between $\beta$ and time to failure $T$, the reliability functions inferred from three statistical models are formulated in following:

Weibull reliability function:

$$R_{\text{Weibull}}(t) = \exp \left( -\left( \frac{1}{t} \lambda_{\text{eff}} \right)^{\beta} \right) = \exp \left( -\frac{1}{t \cdot 1.09849 \times 10^{-3}} \right)^{6.43168}$$  \hspace{1cm} (22)

Lognormal reliability function:

$$R_{\text{Lognormal}}(t) = 1 - \Phi \left( \frac{\ln t - \mu_T}{\eta_T} \right) = 1 - \Phi \left( \frac{\ln t - 11.4968}{0.153694} \right)$$  \hspace{1cm} (23)

Normal reliability function:

$$R_{\text{Normal}}(t) = 1 - \Phi \left( \frac{1/t - \nu_{\text{eff}}}{\phi_{\text{eff}}} \right) = 1 - \Phi \left( \frac{1/t - 1.0280 \times 10^{-5}}{1.6350 \times 10^{-6}} \right)$$  \hspace{1cm} (24)

From the comparison of AIC values calculated by analytical approach, lognormal distribution with lowest AIC value is the best-fit one for the random effect parameter, $\beta$, among the proposed three models. This result is different from the Weibull fitting model for the “pseudo failure time” as shown in the approximate approach. Therefore, there are different statistical properties between random effect parameter and time to failure within the LED’s general degradation path model.

V. CONCLUSIONS

In this paper, two approaches (Approximate approach and Analytical approach) were used to predict the remaining useful lumen life of HPWLED by dealing with the lumen maintenance data. And three types of statistical models (Weibull, Lognormal and Normal) were used to characterize the selected data in both approaches. From the results, we can conclude that:

1) Among the proposed three statistical models, by calculating the AIC values, Weibull model is the best-fitting one to the “pseudo failure time” data in the approximate approach;
2) By the transformation between random effect parameter, $\beta$, and time to failure, $T$, Analytical approach can get the lifetime distributions through fitting the estimated random effect parameter;
3) In analytical approach, Lognormal is the most suitable fitting model for the random effect parameter, $\beta$.

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REFERENCE