

Project Title: Grey Prediction Method used in Failure Prognostics for Electronics

CALCE Team:

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Objectives:

This study is to demonstrate the possibility of using grey prediction model in the failure prognostics for electronics.

Introduction:

Prognostics is a process of predicting the future reliability of the system by assessing the extent of deviation or degradation of a product from its expected normal operating conditions. Failure prognostics for electronics provides data that can be used to meet several critical goals, includes (1) giving advance warning of failures; (2) minimizing unscheduled maintenance, extending maintenance cycles, and maintaining effectiveness through timely repair actions; and (3) reducing the life-cycle cost of equipment by decreasing inspection costs, downtime, and inventory [1].

The grey prediction model, which is part on the grey system theory, can be one step in the failure prognostics approach to perform reliability prediction. Generally speaking, systems can be categorized according to the known knowledge. A black system is a system in which nothing is known about its internal structure, parameters and characteristics. On the contrary, a white system is one in which complete information is known [3]. However, in our daily lives, we often face situations involving incomplete information where some information about a system is knowable and some is not. In this situation, the grey system, which is between the white system and the black one, can be applied.

The grey system theory was developed by Deng [2] in 1982. The main function of it is the effective processing of the analysis, modeling, prediction, decision making and control with incomplete data. The grey prediction model (GM) has been applied in many areas, such as information [11], energy and power [4], industry and economics [9], accident and risk [12], engineering [8], and the environment [3]. In this study, grey prediction was used in failure prognostics of electronics for the first time.

Approach:

The steps used in the grey prediction model are shown in Figure 1 [4]. AGO means the accumulated generating operation, and IAGO means the inverse accumulated generating operation.

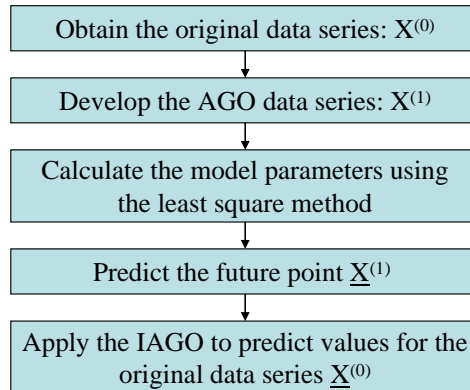


Figure 1. Steps of grey prediction model

The accumulated generating operation (AGO) is used to transform an original set of data into a new set that highlights trends, but has less noise and randomness. The equation used in generating the AGO series is:

$$X^{(1)}(k) = \sum_{i=1}^k X^{(0)}(i) \quad (1)$$

where $X^{(0)}$ represents the original data series, and $X^{(1)}$ represents the AGO series.

After $X^{(1)}$ is obtained, the grey differential equation is built. The general grey differential equation with one variable is:

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = b \quad (2)$$

where $X^{(1)}$ represents the AGO series. The coefficients a and b express the relationship between dX/dt (how fast current state changes) and X (current state).

Parameters a and b are determined using the least-square method, which is shown in the following equations:

$$A = \begin{bmatrix} a \\ b \end{bmatrix} = [\beta^T \beta]^{-1} \beta^T Y \quad (3)$$

where

$$\beta = \begin{bmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -Z^{(1)}(n) & 1 \end{bmatrix} \quad (4)$$

$$Y = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{bmatrix} \quad (5)$$

$$Z^{(1)}(i) = \frac{X^{(1)}(i-1) + X^{(1)}(i)}{2} \quad (6)$$

Then the predicted data points for the AGO series are calculated. $\underline{X}^{(1)}$ represents the predicted AGO series. The equation used in this process is the integral result from equation (2), and can be written as follows:

$$\underline{X}^{(1)}(i+1) = \left(X^{(0)}(1) - \frac{b}{a} \right) e^{-ai} + \frac{b}{a} \quad (7)$$

where $X^{(0)}(1)$ represents the first data in the original series. Then the inverse accumulated generating operation (IAGO) is used to get the inverse data series from AGO. It is then used to transform the forecasted AGO data series back into the original data series. This is achieved using the following equation:

$$\underline{X}^{(0)}(i+1) = \underline{X}^{(1)}(i+1) - \underline{X}^{(1)}(i) \quad (8)$$

where $\underline{X}^{(0)}$ is the predicted original data series. Combine equation (7) and (8), we can get equation (9) as follows:

$$\underline{X}^{(0)}(i+1) = \left(X^{(0)}(1) - \frac{b}{a} \right) (e^{-ai} - e^{-a(i-1)}) \quad (9)$$

To improve prediction accuracy, several researchers presented enhanced grey prediction models, which are discussed in the following paragraphs.

The residual GM model

The residual GM model can be used to minimize prediction error [5]. Some residual errors are observed among the history data and the basic GM model prediction data. The residual GM model is used to treat this residual errors defined by the equation (10) as one series data, and apply the GM model again. The prediction result for this residual error data series is shown in equation (11). Then the residual GM model can be obtained by combining equation (9) and (11), as shown in equation (12):

$$\varepsilon^{(0)}(i) = \left| X^{(0)}(i) - \underline{X}^{(0)}(i) \right| \quad (10)$$

where ε series is the residual error series.

$$\underline{\varepsilon}^{(0)}(i+1) = \left(q^{(0)}(1) - \frac{b'}{a'} \right) \left(e^{-a'i} - e^{-a'(i-1)} \right) \quad (11)$$

$$\underline{X}^{(0)}(i+1) = \left(X^{(0)}(1) - \frac{b}{a} \right) \left(e^{-ai} - e^{-a(i-1)} \right) + \left(\varepsilon^{(0)}(1) - \frac{b'}{a'} \right) \left(e^{-a'i} - e^{-a'(i-1)} \right) \quad (12)$$

The residual GM model shown in equation (12) considers all of the residual errors as positive as defined in equation (10); however, this is not always true. Some residual errors may be negative; therefore it is necessary to look out for signs of a residual error. A comparison was made of different residual GM models including: the residual GM, the Fourier residual GM, the Markov-Chain residual GM, and the artificial neural network (ANN) residual GM. The results showed that the Markov-Chain residual GM yields the best results [9][10][13]. The Markov-Chain model is used to predict the signs of the forecasted residual error data series. If the signs of the residual are positive, it is in state 1; and if the signs are negative, it is in state 2. Each possible transition from state x to state y is associated with a one-step transition probability, P_{xy} as shown in Figure 2. P_{xy} can be estimated using equation (13).

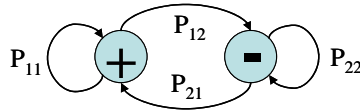


Figure 2. Markov-Chain for residual signs

$$P_{xy} = \frac{M_{xy}}{M_x} \quad (13)$$

where M_x is the number of observations whose residuals are in state x, and M_{xy} is the number of transitions from state x to state y that have occurred. These P_{xy} values can be arranged as a transition matrix R,

$$R = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \quad (14)$$

R can be estimated by examining the signs of residuals from the previous history data.

The initial sign state distribution is denoted by vector $\pi^{(0)} = [\pi_1^{(0)}, \pi_2^{(0)}]$. $\pi^{(0)} = [0, 1]$ represents its initial residual sign as being negative, and $\pi^{(0)} = [1, 0]$ represents its initial residual sign as being positive. The state probabilities after i transitions are given by

$$\pi^{(n)} = \pi^{(0)} R^i \quad (15)$$

where $\pi^{(i)} = [\pi_1^{(i)}, \pi_2^{(i)}]$, if $\pi_1^{(i)} > \pi_2^{(i)}$, the residual sign is positive, and if $\pi_1^{(i)} < \pi_2^{(i)}$, the residual sign is negative. Let the sign δ_i be

$$\delta_i = \begin{cases} +1, & \pi_1^{(i)} > \pi_2^{(i)} \\ -1, & \pi_1^{(i)} < \pi_2^{(i)} \end{cases} \quad (16)$$

So the equation (12) can be updated to

$$\underline{X}^{(0)}(i+1) = \left(X^{(0)}(1) - \frac{b}{a} \right) \left(e^{-ai} - e^{-a(i-1)} \right) + \delta_i \left(q^{(0)}(1) - \frac{b'}{a'} \right) \left(e^{-a'i} - e^{-a'(i-1)} \right) \quad (17)$$

The modified Z series model

It has been found that the basic grey prediction model can be used well with slow growth time sequences, but these often perform poorly and make delay errors for quick growth time sequences [12]. Mao et al. [12] suggested an improved method based on a Z series calculation, as shown in equation (18):

$$Z^{(1)}(k+1) = \frac{1}{2m} [(m+1)x^{(1)}(k) + (m-1)x^{(1)}(k+1)] \quad (18)$$

where

$$m = \left(\sum_{i=2}^n \frac{x^{(1)}(i)}{x^{(1)}(i-1)} \right)^{\frac{1}{n-1}} \quad (19)$$

n is the length of the time sequence window size. This method has proven to be a successful way of widening the adaptability of the grey prediction model to include various kinds of time sequences [12].

Other grey prediction models

Normally, the GM(1,1) model is the default GM(i,j) model. Here i means the order of the differential equation in equation (2), and j means the number of incoming variables. If there is more than one variable in the data series, the GM(1,n) model was introduced by Wu et al. [11]. While high order differential equations can give better prediction results in some situations, they also cost much more computational resources [3]. In addition, a high order differential equation may give imaginary results due to its mathematical calculation, thus the GM(n,1) model is not discussed here in details.

Deliverables:

- This study will demonstrate the possibility and advantages of using grey prediction model in the failure prognostics for electronics.

Project Status:

- The procedure of using grey prediction model in PHM has been developed.

Estimated Schedule:

Tasks	Nov-Dec 2007	Jan-Mar 2008	Apr-June 2008	July-Sept 2008	Oct 2008
Develop the procedure to implement grey prediction model for PHM use.					
Demonstrate the grey prediction model using capacitor failure data.					
Report and review					
		▲			▲

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